

Energy dependent counting statistics in diffusive superconducting tunnel junctions.

P. Samuelsson

Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland.

(Dated: February 1, 2008)

We present an investigation of the energy dependence of the full charge counting statistics in diffusive normal-insulating-normal-insulating-superconducting junctions. It is found that the current in general is transported via a correlated transfer of pairs of electrons. Only in the case of strongly asymmetric tunnel barriers or energies much larger than the Thouless energy is the pair transfer uncorrelated. The second cumulant, the noise, is found to depend strongly on the applied voltage and temperature. For a junction resistance dominated by the tunnel barrier to the normal reservoir, the differential shot noise shows a double peak feature at voltages of the order of the Thouless energy, a signature of an ensemble averaged electron-hole resonance.

PACS numbers: 74.50.+r, 72.70.+m, 74.40.+k

The mechanism for current transport across a normal conductor-superconducting (NS) interface, at energies below the superconducting gap, is Andreev reflection. An electron incident from the normal conductor is retroreflected into a hole, and a charge of $2e$ is transported into the superconductor. The fact that electrons are transferred across the NS-interface in pairs has recently attracted a lot of interest to noise in NS-systems,¹ since the noise can provide information about the charge transfer mechanism.

In some NS-systems, the shot noise is doubled compared to the noise in the corresponding normal system. In a normal-insulating-superconducting (NIS) junction, the pairs tunnel with a small probability between the normal and the superconducting reservoirs. This gives rise to a Poisson noise of pairs of electrons, which is thus twice as large as standard Poisson noise.^{2,3,4} Such pair Poissonian noise was recently observed experimentally.⁵

Also in diffusive wire NS-junctions, a doubling of the shot noise has been predicted theoretically^{3,6,7} and observed experimentally^{8,9,10} for applied voltages much smaller as well as much larger than the Thouless energy. However, for intermediate voltages, of the order of the Thouless energy, the shot noise deviates from the double normal value,^{7,11} a behavior related to the induced proximity effect in the normal conductor.

In general, in mesoscopic NS-systems, there is no simple relation between the shot noise in the normal and the superconducting state. This has been demonstrated for a wide variety of multi-mode systems such as diffusive NIS-junctions,^{3,12} chaotic-dot superconductor junctions^{13,14}, multiterminal superconducting tunnel junctions¹⁵ and wide ballistic double-barrier^{13,16} and normal slab¹⁷ superconductor junctions, as well as few-mode superconducting systems containing beam-splitters,^{18,19,20,21} disordered tunnel junctions²² and double barrier junctions.²³

To obtain complete knowledge of the charge transfer in NS-systems, it is thus not sufficient to study the shot noise. Instead, one has to investigate the full charge counting statistics.²⁴ This was originally done by Muzykanskii and Khmelnitskii,⁴ who showed that the

statistics of the charge transfer across a NS-interface with arbitrary transparency, can in general be described as a correlated transfer of pairs of electrons. Only in the limit of a low transparency NIS junction, the transfer of pairs is uncorrelated, Poissonian. Using a recently developed circuit theory approach^{7,15,25,26} to full counting statistics, the charge transfer statistics has been studied in several systems containing superconductors, such as short SNS-junctions,²⁵ diffusive NS-junctions,⁷ diffusive superconducting tunnel junctions¹⁵ and chaotic dot-superconducting junctions.²⁷ However, in these works, the main focus has been on the low voltage- and temperature properties, the full energy dependence of the counting statistics was only analyzed numerically in Ref. [7].

In this paper we investigate the energy dependent full counting statistics of a diffusive normal-insulating-normal-insulating-superconducting ($NI_1N'I_2S$) junction. This system is interesting for several reasons. Using the circuit theory approach,^{7,15,25} an analytical expression for the full voltage and temperature dependence of the counting statistics can be derived. This allows us to investigate in detail the energy dependence of the charge transfer mechanism. We focus on the role of the proximity effect, which is of particular interest, since it gives rise to a large, energy dependent modification of the transport properties. Moreover, the conductance in $NI_1N'I_2S$ systems was recently studied experimentally,²⁸ and very good agreement was found between the experiment and the quasiclassical Greens function theory²⁹ on which the circuit theory is based. This makes a detailed analysis of the experimentally accessible noise of interest.

We find that the current is in general transported via a correlated transfer of pairs of electrons. However, in the cases where the resistance is dominated by the tunnel barrier I_2 , at the $N'I_2S$ interface, or for energies well above the Thouless energy, the pair transfer is uncorrelated. These cases correspond to the limit of weak proximity effect in the normal region N' . In the opposite regime of strong proximity effect in N' , when the resistance is dominated by the tunnel barrier I_1 at the NI_2N' interface, a proximity gap opens up in the N' -region. For energies below the induced proximity gap, the junction is

effectively a NIS-junction and the pair transfer is again uncorrelated.

The second cumulant, the noise, is found to depend strongly on the applied voltage and temperature. For a junction resistance dominated by the tunnel barrier I_1 , at the NI_1N' interface, and a temperature well below the Thouless energy, the differential noise shows a double-peak feature at voltages of the order of the Thouless energy. We show that this double-peak behavior can be attributed to an ensemble averaged electron-hole resonance.

The paper is organized as follows. We first present the circuit theory model and the derivation of the full charge counting statistics. Thereafter, the dependence of the counting statistics on energy and tunnel barrier (I_1 and I_2) conductances is analyzed. We then investigate the temperature and voltage dependence of the noise in detail.

I. MODEL AND THEORY

We consider a diffusive normal-insulating-normal-insulating-superconducting ($NI_1N'I_2S$) junction (see Fig. 1), where both the resistances of the insulating tunnel barriers (I_1 and I_2) are much larger than the resistance of the normal diffusive region (N') between the barriers. The short diffusive region N' , which does not contribute to the junction resistance, leads to complete isotropization of the system parameters between the barriers.³⁰ This makes the junction effectively zero dimensional³¹ and the exact shape and size of the diffusive region is thus not important.

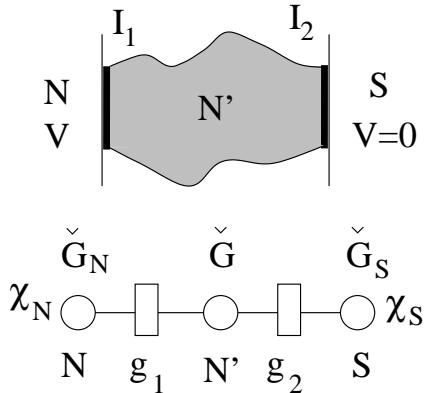


FIG. 1: The $NI_1N'I_2S$ -junction. Upper figure: A normal (N) and a superconducting (S) reservoir are connected via a diffusive normal metal (N' , shaded) contact. At the NN' and $N'S$ interface, there are tunnel barriers I_1 and I_2 with conductance g_1 and g_2 respectively. Lower figure: The circuit theory representation of the junction, with the matrix Greens functions \check{G}_N , \check{G}_S and \check{G} and the counting fields χ_N and χ_S shown.

The conductances g_1 and g_2 of the two tunnel barriers can be written as $g_{1(2)} = (2e^2/h) \sum_{n=1}^{N_{1(2)}} \Gamma_{1(2),n}$, where

$N_{1(2)}$ is the number of transport modes at the barrier $1(2)$ and $\Gamma_{1(2),n}$ is the transparency of mode n . We consider the limit of large number of modes, $N_{1(2)} \gg 1$ and small transparency of each mode, $\Gamma_{1(2),n} \ll 1$. However, the conductances g_1 and g_2 are much larger than the conductance quanta $2e^2/h$, i.e. $\sum_{n=1}^{N_{1(2)}} \Gamma_{1(2),n} \gg 1$. Under these conditions we can neglect Coulomb blockade effects as well as weak localization like corrections to the cumulants of the current.

It is further assumed that the dwell time of the particles in the the normal region N' is much shorter than the inelastic scattering- and phase breaking times. We are also only interested in zero frequency properties.³² With these assumptions, we can analyze the full counting statistics of the junction within the framework of the circuit theory, recently developed by Nazarov^{30,33} and Belzig and Nazarov.^{7,25} Very recently, a detailed discussion of the circuit theory was given.²⁶ We note that the zero energy counting statistics was studied for the same junction in Ref. [15], here we study the full energy dependence.

In the circuit theory, the mesoscopic conductor is described in terms of nodes connected via various circuit elements, just as in an ordinary electric circuit theory. Due to the effective zero-dimensionality of the junction, the circuit theory representation (see Fig. 1) only contains three nodes, the two reservoirs (N and S) and the isotropic normal region (N') between the tunnel barriers, and two circuit elements, the tunnel barriers (I_1 and I_2).

However, instead of characterizing each node with an electric potential, as is done in an ordinary electrical circuit theory, they are characterized with a 4×4 matrix Green's function (in Keldysh-Nambu space). Here we have the three node Green's functions \check{G}_N , \check{G}_S and \check{G} , as shown in Fig. 1. They obey the normalization condition $\check{G}_N^2 = \check{G}_S^2 = \check{G}^2 = \check{1}$. Throughout the paper we use the convention with “check”, $\check{\cdot}$ denoting 4×4 matrices and “hat”, $\hat{\cdot}$ denoting 2×2 matrices.

The matrices of the reservoir nodes are known. In the normal reservoir we have

$$\check{G}_N = e^{i\chi_N \check{\tau}_K/2} \check{G}_N^0 e^{-i\chi_N \check{\tau}_K/2}, \quad \check{\tau}_K = \begin{pmatrix} 0 & \hat{\sigma}_z \\ \hat{\sigma}_z & 0 \end{pmatrix} \quad (1)$$

$$\check{G}_N^0 = \begin{pmatrix} \hat{\sigma}_z & \hat{K}_N \\ 0 & -\hat{\sigma}_z \end{pmatrix}, \quad \hat{K}_N = 2 \begin{pmatrix} 1 - 2f_+ & 0 \\ 0 & 2f_- - 1 \end{pmatrix},$$

where $f_{\pm}(E) = f(E \pm eV)$, with $f(E) = (\exp[E/kT] + 1)^{-1}$ the Fermi distribution function. Here χ_N is a counting field, “counting” the electrons passing in and out of the normal reservoir and $\hat{\sigma}_z$ is a Pauli matrix. In the superconducting reservoir we have

$$\check{G}_S = e^{i\chi_S \check{\tau}_K/2} \check{G}_S^0 e^{-i\chi_S \check{\tau}_K/2}$$

$$\check{G}_S^0 = \begin{pmatrix} \hat{R}_S & \hat{K}_S \\ 0 & \hat{A}_S \end{pmatrix}, \quad \hat{R}(\hat{A})_S = \begin{pmatrix} g_{R(A)} & f_{R(A)} \\ f_{R(A)} & -g_{R(A)} \end{pmatrix},$$

$$\hat{K}_S = (\hat{R}_S - \hat{A}_S)[f(E) - f(-E)], \quad (2)$$

where Δ is the superconducting gap and $f_{R(A)} = i\Delta[(E \pm$

$i\epsilon)^2 - \Delta^2]^{-1/2}$ and $g_{R(A)} = [1 - f_{R(A)}^2]^{1/2}$ are Green's functions characterizing the superconducting reservoir (ϵ is an infinitesimal positive number). The counting field χ_S of the superconducting reservoir is introduced for notational clarity, due to current conservation it is in principle not needed.

The matrix \check{G} of the node between the tunnel barriers is to be determined. It is found from a conservation law for matrix currents (a matrix Kirchoff's rule), as

$$\check{I}_1 + \check{I}_2 + \check{I}_E = 0 \quad (3)$$

where the matrix currents are given by

$$\begin{aligned} \check{I}_1 &= \frac{g_1}{2} [\check{G}_N, \check{G}], \quad \check{I}_2 = \frac{g_2}{2} [\check{G}_S, \check{G}] \\ \check{I}_E &= -\frac{2iE}{\delta} [\check{G}_E, \check{G}], \quad \check{G}_E = \begin{pmatrix} \hat{\sigma}_z & 0 \\ 0 & \hat{\sigma}_z \end{pmatrix} \end{aligned} \quad (4)$$

where $[A, B] = AB - BA$ and δ is the mean level spacing in the N' -region divided by the conductance quanta $2e^2/h$. The Thouless energy is given by $E_{Th} = (g_1 + g_2)\delta/4$, i.e. it depends only on the escape time through the tunnel barriers, not on the diffusion time through the N' -region. The term \check{I}_E leads to a modification of the proximity effect at finite energies, and eventually a suppression for $E \gg E_{Th}$.

From Eq. (3) and the condition $\check{G}^2 = 1$, we can determine the missing node matrix $\check{G}(\chi_N, \chi_S)$ as a function of the counting fields. Following the lines of Ref. [15], noting that Eq. (3) can be written in the form $[\check{G}, \check{G}_x] = 0$, we find the physically relevant solution³⁴

$$\begin{aligned} \check{G} &= \check{G}_x (\check{G}_x^2)^{-1/2}, \\ \check{G}_x &= \frac{g_1}{2} \check{G}_N + \frac{g_2}{2} \check{G}_S - i \frac{E}{2E_{Th}} (g_1 + g_2) \check{G}_E. \end{aligned} \quad (5)$$

Knowing $\check{G}(\chi_N, \chi_S)$, we can find the cumulant generating function $F(\chi_N, \chi_S)$. It is given from the general relation between the matrix current and the generating function,³³

$$\begin{aligned} \frac{\partial F(\chi_N, \chi_S)}{\partial \chi_N} &= \frac{it_0}{8e^2} \int dE \operatorname{tr} [\check{\tau}_K \check{I}_1(\chi_N, \chi_S)] \\ \frac{\partial F(\chi_N, \chi_S)}{\partial \chi_S} &= \frac{it_0}{8e^2} \int dE \operatorname{tr} [\check{\tau}_K \check{I}_2(\chi_N, \chi_S)] \end{aligned} \quad (6)$$

where t_0 is the measurement time. Using that under the trace,

$$\begin{aligned} \frac{\partial (\check{G}_x^2)^{1/2}}{\partial \chi_{N(S)}} &= \frac{\partial \check{G}_x}{\partial \chi_{N(S)}} \check{G}_x (\check{G}_x^2)^{-1/2} = \frac{i}{4} g_{1(2)} [\check{\tau}_K, \check{G}_{N(S)}] \check{G} \\ &= \frac{i}{4} g_{1(2)} \check{\tau}_K [\check{G}_{N(S)}, \check{G}] = \frac{i}{2} \check{\tau}_K I_{1(2)}, \end{aligned} \quad (7)$$

we get the counting field dependent part of the cumulant generating function

$$F(\chi_N, \chi_S) = \frac{t_0}{4e^2} \int dE \operatorname{tr} [(\check{G}_x^2)^{1/2}]. \quad (8)$$

We note that the standard definition of a generating function demands $F(\chi_N = 0, \chi_S = 0) = 0$. This implies that a counting field independent constant $-\operatorname{tr}[\check{G}_x^2(\chi_N = 0, \chi_S = 0)]^{1/2}$ should in principle be added to the integrand in Eq. (8). However, such a constant does not alter the cumulants, and below we will for simplicity omit it.

The probability $P(Q)$ that Q electron charges have been transported through the junction in time t_0 is given by

$$P(Q) = \int_{-\pi}^{\pi} \frac{d\chi_N}{2\pi} e^{-i\chi_N Q - F(\chi_N, \chi_S=0)} \quad (9)$$

Eqs. (2) - (9) completely determine the full charge counting statistics for arbitrary temperatures and voltages. For our purposes, it is however more convenient to focus on the properties of the cumulant generating function in Eq. (8). From the cumulant generating function we can derive all cumulants of the current by repeatedly taking the derivative with respect to the counting field χ_N (or χ_S). For the two first cumulants, current and noise, we have (evaluated at the N -contact),

$$I = i \frac{e}{t_0} \frac{\partial F(\chi_N, \chi_S)}{\partial \chi_N} \Big|_{\chi_N=\chi_S=0}, \quad (10)$$

$$S = -\frac{2e^2}{t_0} \frac{\partial^2 F(\chi_N, \chi_S)}{\partial \chi_N^2} \Big|_{\chi_N=\chi_S=0}. \quad (11)$$

and similarly for higher order cumulants.

II. FULL COUNTING STATISTICS.

In what follows we focus on the limit $kT, eV, E_{Th} \ll \Delta$. In this limit, the physics is dominated by the induced proximity effect in the diffusive region N' between the two barriers.³⁵ The proximity effect affects the transport properties on the energy scale E_{Th} . It can on a microscopic level be described as resulting from coherence between the two quasiparticles in the Andreev reflection, e.g. an incident electron and a retroreflected hole. An induced proximity effect leads in general to large modifications of the transport properties, i.e. modifications scaling with the system size. We note that this is in strong contrast to the situation in normal systems, where phase coherence only gives rise to small quantum corrections.¹

However, being a coherent phenomenon, the proximity effect is also sensitive to modifications of the electron and hole phases. At finite energies, the electron and hole pick up different phases and at $E \gg E_{Th}$, the electron-hole coherence is destroyed and consequently, the proximity effect is suppressed. Moreover, a magnetic flux in the N' -region large enough to break time reversal symmetry, also suppresses the proximity effect, since the electron and the hole pick up different phases in a finite magnetic field. Here we however only consider the case with negligibly small magnetic fields.

Formally, at energies well below the superconducting gap Δ , the matrix Greens function \tilde{G}_S^0 in Eq. (2) simplifies considerably, since $\hat{R}_S = \hat{A}_S = \hat{\sigma}_x$ and consequently $\hat{K}_S = 0$. We then find $F(\chi_N, \chi_S)$ in Eq. (8) by diagonalizing $(\tilde{G}_x^2)^{1/2}$, giving

$$F(\chi_N, \chi_S) = \frac{t_0}{e^2} \int \frac{dE}{2\sqrt{2}} \sqrt{\alpha + \sqrt{\alpha^2 + \beta^2 - 4g_1^2 g_2^2 \Lambda}}, \quad (12)$$

where we have introduced

$$\begin{aligned} \alpha &= g_1^2 + g_2^2 - \frac{E^2}{E_{Th}^2} (g_1 + g_2)^2, \\ \beta &= 2g_1(g_1 + g_2) \frac{E}{E_{Th}}, \\ \Lambda &= \left(1 - e^{2i(\chi_N - \chi_S)}\right) f_+(1 - f_-) \\ &+ \left(1 - e^{-2i(\chi_N - \chi_S)}\right) f_-(1 - f_+). \end{aligned} \quad (13)$$

We note that at $E = 0$, we recover the generating function of Ref. [15]. The terms $\exp(\pm 2i[\chi_N - \chi_S])$ show that electrons are transported in pairs out of (+) and into (-) the superconductor and into (+) and out of (-) the normal reservoir. This pair transport follows from the fact that the charge transfer mechanism across the NS -interface is Andreev reflection. The form of the cumulant generating function, with the double square roots, tells that the pair transfer is highly correlated. There are however several limits where the generating function is simply proportional to Λ , describing an uncorrelated, (generalized) Poissonian, transfer of pairs of electrons (as follows from Eq. 9).

In the limit of high energy, $E \gg E_{Th}$, for arbitrary g_1 and g_2 , the integrand of the generating function in Eq. (12) takes on the form

$$\frac{\partial F(\chi_N, \chi_S)}{\partial E} = \frac{t_0}{4e^2} \frac{g_1 g_2^2}{(g_1 + g_2)^2} \left(\frac{E_{Th}}{E} \right)^2 \Lambda. \quad (14)$$

For these energies, the probability of pair tunneling is strongly suppressed and the pairs are thus emitted in a Poisson process. This is a consequence of the different phases picked up by the electrons and holes, $\sim \pm E/E_{Th}$, at finite energies. At $E \gg E_{Th}$ the electrons and holes loose their coherence and the pair tunneling probability is suppressed.

In the limit of a dominating coupling of the diffusive region N' to the normal reservoir, $g_1 \gg g_2$, the integrand of the generating function becomes

$$\frac{\partial F(\chi_N, \chi_S)}{\partial E} = \frac{t_0}{4e^2} \frac{g_2^2}{g_1} \frac{1}{1 + (E/E_{Th})^2} \Lambda. \quad (15)$$

Thus, in this limit, the pair transfer is also uncorrelated. In this case, the barrier I_1 has a negligible resistance compared to I_2 and the junction is effectively a NI_2S -junction. As discussed above, in this case the pair

transfer is uncorrelated. We note that for high energies $E \gg E_{Th}$, it coincides with Eq. (14), (for $g_1 \gg g_2$).

We also note¹⁵ that for the limit $g_1 \ll g_2$, strong coupling of N' to the superconductor and for the additional limit of low energies, $E \ll E_{Th}$, the pair transfer is uncorrelated. In this case, the integrand of the generating function is simply

$$\frac{\partial F(\chi_N, \chi_S)}{\partial E} = \frac{t_0}{4e^2} \frac{g_1^2}{g_2} \Lambda \quad (16)$$

The reason for the uncorrelated pair transfer in this case is the following: In the limit $g_1/g_2 \rightarrow 0$, a proximity gap opens up in N' and the junction becomes an effective NI_1S' junction (S' denoting the gapped normal region between the barriers). For energies below the induced gap, the pair transfer is thus Poissonian, as in a standard NIS -junctions.^{2,3,4}

Making a connection to the proximity effect, it can be noted that in two cases, Eq. (14) and Eq. (15), the strongly suppressed proximity effect in N' is responsible for the uncorrelated pair transport. However, in the last case, Eq. (16), the Poissonian pair transfer is due to the induced proximity gap in N' .

It is instructive to also study qualitatively the full counting statistics, starting from the scattering point of view in Ref. [4]. The cumulant generating function, generalized to many modes, is

$$F(\chi_N, \chi_S) = \frac{t_0}{h} \int dE \langle \ln(1 + R_{eh}(E)\Lambda) \rangle \quad (17)$$

where the $R_{eh}(E)$'s are the Andreev reflection eigenvalues, i.e. the real and energy dependent eigenvalues of the hermitian electron-hole scattering matrix product $S_{eh}^\dagger S_{eh}$ (see e.g. Ref. [13]). Here $\langle \dots \rangle$ denotes ensemble average over impurity configurations and Λ is the same as in Eq. (13). For a general function $h(R_{eh})$, the ensemble average can be written as an integration $\int dR_{eh} \rho(R_{eh}) h(R_{eh})$, where $\rho(R_{eh})$ is the distribution of Andreev reflection eigenvalues (the corresponding quantity in NS -systems to the transmission eigenvalue distribution in normal systems).

Expanding the logarithm in Eq. (17), makes it is clear that the limits where the charge transfer is uncorrelated, Eqs. (14) to (16), corresponds to the limit where $\langle R_{eh} \rangle \gg \langle R_{eh}^2 \rangle$ (and consequently $\langle R_{eh} \rangle \gg \langle R_{eh}^n \rangle$ for all higher moments $n > 2$ since $0 \leq R_{eh} \leq 1$).

From this we can draw the following conclusions. The limits of uncorrelated pair transfer corresponds to the cases where there are no or very few “open” Andreev channels, i.e with Andreev reflection probability close to unity (leading to $\langle R_{eh} \rangle \gg \langle R_{eh}^2 \rangle$). In the case where there are more open channels, the various moments $\langle R_{eh}^n \rangle$ are of the same order of magnitude, resulting e.g. in a noise below the Poissonian noise of uncorrelated pair transfer across the NS -interface, further discussed below. As is also further discussed below, this Andreev channel picture provides a simple explanation for the voltage and

temperature dependence of the noise as well as the current.

III. NOISE.

The rest of the paper is devoted to a detailed analysis of the properties of the the second cumulant, the noise, which is within reach of existing experimental techniques. We note that the cumulant generating function $F(\chi_N, \chi_S)$ in Eq. (12) can be written as g_1 (or g_2) times a function which only contains the ratio g_1/g_2 , i.e. all transport properties studied below, (normalized) depend only on the ratio of the conductances.

For the benefit of the following discussion, we first study the current, given from Eq. (10) and (12),

$$I = \frac{1}{e} \int dE \frac{f_- - f_+}{\sqrt{2}} \frac{g_1^2 g_2^2}{\sqrt{\alpha^2 + \beta^2} \sqrt{\alpha + \sqrt{\alpha^2 + \beta^2}}} \quad (18)$$

The current has been studied both theoretically^{29,30} and experimentally,⁵ here we briefly summarize the findings. The differential conductance $G = dI/dV$ at zero temperature is plotted as a function of eV/E_{Th} in Fig. 2, for different ratios g_1/g_2 . The conductance for $eV \ll E_{Th}$ is

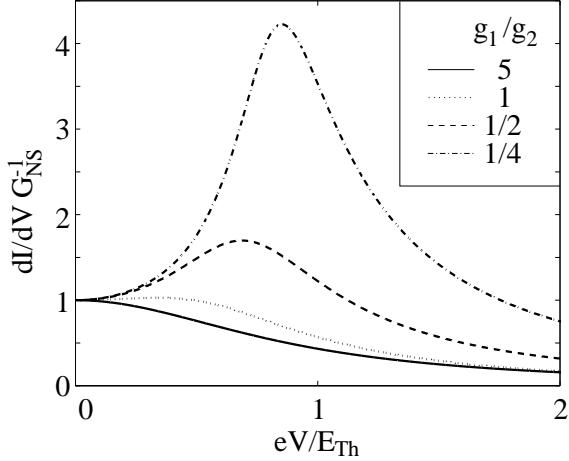


FIG. 2: The differential conductance dI/dV , normalized with the zero voltage conductance G_{NS} , as a function of eV/E_{Th} . The values for the ratios $g_1/g_2 = 5$ and the corresponding line types are shown in the legend. The temperature $kT \ll eV$. For $g_2 = 4g_1$, the conductance shows a strong peak at $eV \sim E_{Th}$, a signature of an ensemble averaged electron-hole resonance.

given by

$$G(eV \ll E_{Th}) \equiv G_{NS} = \frac{g_1^2 g_2^2}{(g_1^2 + g_2^2)^{3/2}}. \quad (19)$$

In the opposite limit, $eV \gg E_{Th}$, the conductance decays as a power law with applied voltage, as $(E_{Th}/eV)^2 g_2^2 g_1 / (g_1 + g_2)^2$.

The behavior of the conductance for voltages of the order of E_{Th} depends strongly on the ratio g_1/g_2 . For $g_1 \gg g_2$, i.e. for a strong coupling of the normal region N' to the normal reservoir, the conductance decreases monotonically with voltage as $(g_2^2/g_1)/[1 + (eV/E_{Th})^2]$. In the opposite regime, $g_2 \gg g_1$, the normal region is strongly coupled to the superconducting reservoir. The conductance has a strong peak at voltages $eV \sim E_{Th}$, which can be seen as an ensemble averaged electron-hole resonance. In other words, for energies $E \sim E_{Th}$ the distribution of Andreev reflection eigenvalues, $\rho(R_{eh})$, is shifted towards more “open” channels.

As pointed out above, in the limit of $g_1/g_2 \rightarrow 0$, i.e. when decoupling the normal reservoir, a proximity induced gap E_{Th} opens up in the spectrum in the normal region N' . In this limit, the conductance shows a singularity at $eV = E_{Th}$, similar to the standard NIS-tunneling conductance, but with the induced gap E_{Th} instead of the superconducting gap Δ .

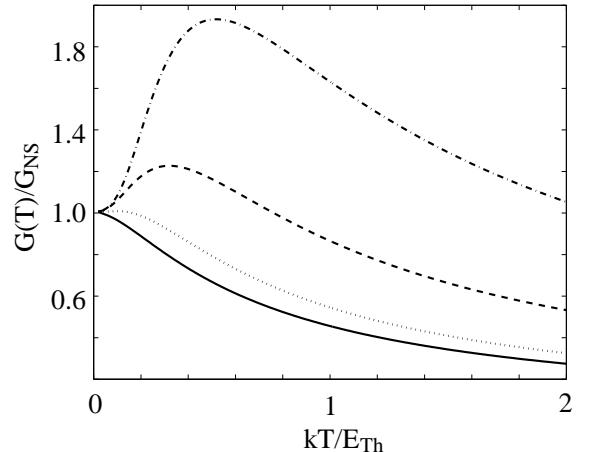


FIG. 3: The zero voltage conductance normalized with G_{NS} , as a function of temperature. The values for g_1/g_2 and the corresponding line-types are the same as in Fig. 2. The conductance shows a temperature dependence which is qualitatively similar to the voltage dependence of the conductance in Fig. 2. The zero voltage conductance is related to the equilibrium noise as $S_{eq}(T) = 4ekTG(eV = 0, T)$.

The temperature dependence of the zero voltage conductance, $G(T)$, is shown in Fig. 3. The temperature dependence is qualitatively similar to the voltage dependence, shown in Fig. 2, due to the form of the spectral current density in Eq. (18). We note that for $kT \ll E_{Th}$ we have $G = G_{NS}$ from Eq. (19). In the opposite limit, $kT \gg E_{Th}$ the conductance decays as $1/kT$. In the intermediate regime, the behavior of the conductance depends strongly on the ratio g_1/g_2 . For $g_1 \gg g_2$, it decays monotonically with temperature, while in the opposite limit, $g_2 \gg g_1$, it has a peak at temperatures kT of order E_{Th} .

We then turn to the noise, found from Eq. (11) and

(12) to be

$$S = \int dE \frac{2\sqrt{2}g_1^2g_2^2}{\sqrt{\alpha^2 + \beta^2} \left(\alpha + \sqrt{\alpha^2 + \beta^2} \right)^{3/2}} \times \left[[f_+(1-f_-) + f_-(1-f_+)] \left(\alpha + \sqrt{\alpha^2 + \beta^2} \right) - (f_- - f_+)^2 \frac{g_1^2g_2^2}{\alpha^2 + \beta^2} \left(2\alpha + 3\sqrt{\alpha^2 + \beta^2} \right) \right]. \quad (20)$$

First we consider the equilibrium noise, for $eV \ll E_{Th}, kT$. In this limit we have $f_- - f_+ = 0$ and the noise is

$$S_{eq} = \int dE \frac{4\sqrt{2}f(1-f)g_1^2g_2^2}{\sqrt{\alpha^2 + \beta^2} \sqrt{\alpha + \sqrt{\alpha^2 + \beta^2}}}. \quad (21)$$

Noting that we have $2f(1-f) = kT[d(f_- - f_+)/dV]_{eV=0} = -2kT(df/dE)$ we see from Eq. (18) and (21) that

$$S_{eq}(T) = 4kTG(eV = 0, T), \quad (22)$$

for arbitrary temperature T . This is just what we expect from the fluctuation-dissipation theorem. The temperature dependence of the noise, normalized with $4kT$, is thus just the temperature dependence of the zero voltage conductance, plotted in Fig. (3).

At $kT \ll E_{Th}$ we find just $S_{eq}(kT \ll E_{Th}) = 4kTG_{NS}$. In the opposite limit, $kT \gg E_{Th}$ the equilibrium noise tends towards a constant, temperature independent value

$$S_{eq}(kT \gg E_{Th}) = E_{Th} \frac{\pi g_1 g_2^2}{(g_1 + g_2) \sqrt{g_1^2 + g_2^2}}. \quad (23)$$

In the intermediate temperature regime, the noise, as the conductance in Fig. 3, depends strongly on the ratio g_1/g_2 .

The voltage dependence of the noise is qualitatively different from the temperature dependence. In the limit $kT \ll eV, E_{Th}$, the shot noise regime, we have

$$S = \int_{-eV}^{eV} dE \frac{\sqrt{2}g_1^2g_2^2}{\left(\alpha + \sqrt{\alpha^2 + \beta^2} \right)^{3/2}} \times \left(-1 + \frac{3g_1^2g_2^2}{\alpha^2 + \beta^2} - \frac{\alpha(\alpha^2 + \beta^2 - 2g_1^2g_2^2)}{(\alpha^2 + \beta^2)^{3/2}} \right). \quad (24)$$

Here we focus on the differential noise, dS/dV , which is plotted as a function of voltage for different ratios g_1/g_2 in Fig. 4.

In the low voltage limit, $eV \ll E_{Th}$, the differential noise is given by^{13,15}

$$\left. \frac{dS}{dV} \right|_{eV \ll E_{Th}} \equiv P_{NS} = 2eG_{NS} \left(2 - \frac{5g_1^2g_2^2}{(g_1^2 + g_2^2)^2} \right). \quad (25)$$

The Fano factor, $P_{NS}/2eG_{NS}$, in this limit, is thus $(2 - 5g_1^2g_2^2/[g_1^2 + g_2^2]^2)$. It varies from 2 in the limits

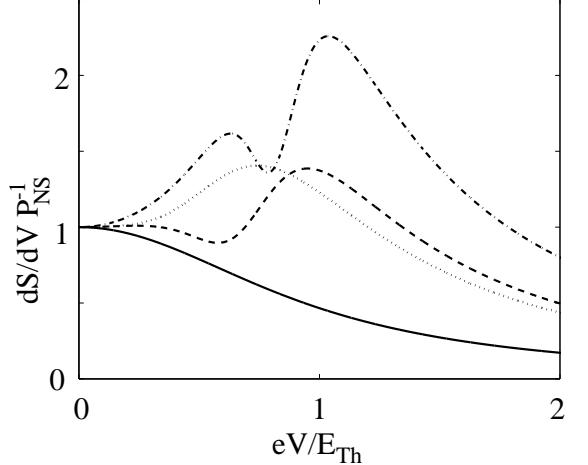


FIG. 4: The differential shot noise dS/dV as a function of applied voltage, normalized with the zero voltage value. The values for g_1/g_2 and the corresponding line-types are the same as in Fig. 2. The temperature $kT \ll E_{Th}$. The double-peak behavior at $eV \sim E_{Th}$, for $g_2 = 4g_1$, is a signature of the electron-hole resonance, responsible for the peak in the conductance at $eV \sim E_{Th}$ in Fig. 2.

$g_1 \gg g_2$ and $g_2 \gg g_1$, to $3/4$ for¹⁶ $g_1 = g_2$. A Fano factor 2 is an indication of uncorrelated emission of pairs of electrons into or out of the superconductor. As pointed out in connection to Eq. (15) and (16), in these limits, for $eV \ll E_{Th}$ and $g_1 \gg g_2$ or $g_2 \gg g_1$, the generating function $F(\chi_N, \chi_S)$ in Eq. (12) is proportional to¹⁵ $\exp(2i[\chi_N - \chi_S])$, showing that the charge transfer is indeed an uncorrelated emission (for negative voltages) of pairs of electrons from the superconductor.

In the high voltage limit, $eV \gg E_{Th}$, the differential noise decreases with voltage as

$$\frac{1}{e} \frac{dS}{dV} = 2 \left(\frac{E_{th}}{eV} \right)^2 \frac{g_2^2 g_1}{(g_1 + g_2)^2} \quad (26)$$

i.e. the same power law behavior as the conductance, which is clear directly from the generating function in Eq. (14).

At intermediate voltages, the noise depends strongly on the ratio g_1/g_2 . For a dominating coupling of the normal region to the normal reservoir, $g_1 \gg g_2$, the differential noise decreases monotonically with applied voltage, as

$$\frac{1}{e} \frac{dS}{dV} = 2 \frac{g_2^2}{g_1} \frac{1}{1 + (eV/E_{Th})^2}. \quad (27)$$

This can also be obtained directly from the generating function in Eq. (15). We note that this behavior is very similar to what was found for a diffusive *NIS*-junction,¹² a diffusive normal region of non-negligible resistance, connected to a superconducting reservoir via a tunnel barrier.

In the opposite limit, $g_1 \gg g_2$, the noise shows a double peak structure around $eV \sim E_{Th}$, the voltage where

the conductance shows a single peak (see Fig. 2). Such a double peak behavior has been discussed before in single mode junctions with sharp Andreev resonances.¹⁶ In single mode junctions, the double peak behavior follows directly from the fact that if the Andreev reflection probability R_{eh} has a resonance at some energy E_0 , the noise, $\sim R_{eh}(1 - R_{eh})$, shows a double peak around E_0 . Interestingly, as is clear from Fig. 4, in the $NI_1N'I_2S$, this double-peak behavior survives the ensemble average. This indicates that the whole distribution $\rho(R_{eh})$ of Andreev reflection eigenvalues is shifted from being dominated by “closed” towards being dominated by “open” channels at resonance $E \sim E_{Th}$.

We note that such a double peak behavior can also be found in normal single-mode double-barrier junctions, it is not an effect of the superconductivity (see Ref. [1]). However, in normal, many-mode systems, such a behaviour is, to leading order in number of modes, washed out when performing an ensemble average.

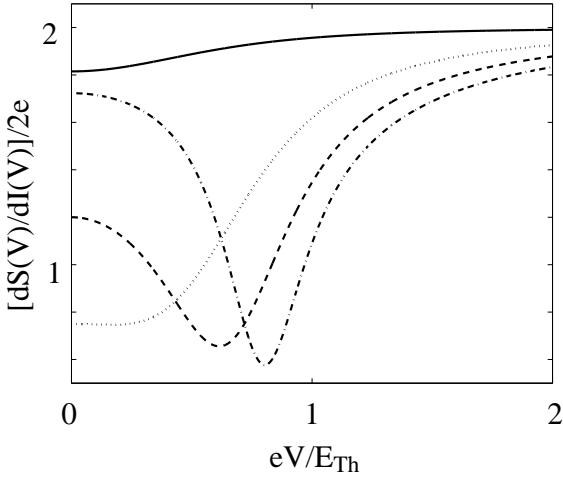


FIG. 5: The differential Fano factor $[dS(V)/dI(V)]/2e$ as a function of applied voltage. The values for g_1/g_2 and the corresponding line-types are the same as in Fig. 2. The temperature $kT \ll E_{Th}$. The dip at $eV \sim E_{Th}$, for $g_2 = 4g_1$, is a signature of the electron-hole resonance, responsible for the peak in the conductance at $eV \sim E_{Th}$ in Fig. 2.

Additional insight can be obtained by studying the differential Fano factor, $[dS(V)/dI(V)]/2e$. In the differential Fano factor, the voltage dependence of the conductance is canceled out from the noise, and we obtain the “bare” voltage-dependence of the noise. We have

$$\frac{1}{2e} \frac{dS(V)}{dI(V)} = 2 - \frac{2g_1^2 g_2^2 (2\alpha + 3\sqrt{\alpha^2 + \beta^2})}{(\alpha^2 + \beta^2) (\alpha + \sqrt{\alpha^2 + \beta^2})}. \quad (28)$$

This quantity is plotted in Fig. 5. We note that the differential Fano factor shows a much stronger dependence of the applied voltage, compared to earlier studied^{7,11,12,36,37} diffusive NS and NIS junctions. The resonant double-peak in the noise, for $g_2 \gg g_1$ is not visible in the differential Fano factor, instead it shows a dip at $eV \sim E_{Th}$. This is in accordance with our explanation of the double peak-behavior of the noise above, since, again making the comparison to a single mode junction, the differential Fano factor is the differential shot noise divided by the differential conductance, $\sim [R_{eh}(1 - R_{eh})]/R_{eh} = 1 - R_{eh}$, which thus has a dip at the resonant energy.

Moreover, in the high voltage limit, $eV \gg E_{Th}$, the differential Fano factor saturates at a constant value 2, i.e. independent on the relation between the conductances g_1 and g_2 . This is a manifestation of the uncorrelated charge transfer at these energies, as discussed in connection to Eq. (14). Also in the limit $g_2 \gg g_1$, the Fano factor approaches 2, independent on voltage. This is again a manifestation of the uncorrelated pair transfer, and can be seen directly from the generating functions in Eq. (15). From this we conclude that the differential Fano factor is thus the relevant quantity for studying the energy dependence of the charge transfer mechanism.

IV. CONCLUSIONS

In conclusion, we have investigated the energy dependent full counting statistics of a diffusive normal-insulating-normal-insulating-superconducting ($NI_1N'I_2S$) junction. We have used the recently developed circuit theory of full counting statistics, allowing us to access the full temperature and voltage dependence of the statistics. In general, the charge is transported via correlated transfer of pairs of electrons. However, in the case of strongly asymmetric contacts or energies much larger than the Thouless energy, the pair transfer is uncorrelated. The second cumulant, the noise, was studied in detail. It was found to depend strongly on voltage and temperature. For low temperatures and a junction resistance dominated by the tunnel barrier to the normal reservoir (I_1), the noise shows a double-peak behavior at voltages $eV \sim E_{Th}$, a signature of an ensemble averaged electron-hole resonance.

We acknowledge discussions with F. Lefloch, W. Belzig and M. Büttiker. This work was supported by the Swiss National Science Foundation and the project for Materials with Novel Electronic Properties.

¹ For a review, see Ya. M. Blanter and M. Büttiker, Phys. Rep. **336** 1 (2000).

² V.A. Khlus, Sov. Phys. JETP **66** 1243, (1987).

³ M.J.M. de Jong and C.W.J. Beenakker, Phys. Rev. B **49** 16070, (1994).

⁴ B.A. Muzykantskii, and D.E. Khmelnitskii, Phys. Rev. B

- ⁵ **50** 3982, (1994).
- ⁵ F. Lefloch, C. Hoffman, M. Sanquer and D. Quirion, cond-mat/0208126.
- ⁶ K.E. Nagaev and M. Büttiker, Phys. Rev. B **63** 081301, (2001).
- ⁷ W. Belzig and Yu.V. Nazarov, Phys. Rev. Lett. **87** 067006 (2001).
- ⁸ X. Jehl, M. Sanquer, R. Calemczuk, and D. Mailly, Nature (London) **405** 50 (2000).
- ⁹ A.A. Kozhevnikov, R.J. Schoelkopf, and D.E. Prober, Phys. Rev. Lett. **84** 3398 (2000).
- ¹⁰ X. Jehl, and M. Sanquer, Phys. Rev. B **63** 052511 (2001).
- ¹¹ B. Reulet, A.A. Kozhevnikov, D.E. Prober, W. Belzig, and Yu. V. Nazarov, condmat/0208089.
- ¹² M.P.V. Stenberg, and T. Heikkilä, condmat/0208235.
- ¹³ M.J.M. de Jong and C.W.J. Beenakker, in: Mesoscopic Electron Transport, edited by L.L. Sohn, L.P. Kouwenhoven, and G. Schön , NATO ASI Series E345 (Kluwer, Dordrecht, 1997).
- ¹⁴ P. Samuelsson and M. Büttiker, Phys. Rev. Lett. **89** 046601 (2002).
- ¹⁵ J. Börlin, W. Belzig and C. Bruder, Phys. Rev. Lett. **88**, 197001 (2002).
- ¹⁶ A.L. Fauchere, G.B. Lesovik, and G. Blatter, Phys. Rev. B **58** 11177, (1998).
- ¹⁷ M. Schechter, Y. Imry and Y. Levinson, Phys. Rev. B **64** 224513 (2001).
- ¹⁸ M. P. Anantram and S. Datta, Phys. Rev. B **53** 16390 (1996).
- ¹⁹ Th. Martin, Phys. Lett. A **220** 137 (1996).
- ²⁰ J. Torrés and Th. Martin, Eur. Phys. J. B **12** 319 (1999).
- ²¹ F. Taddei and R. Fazio, Phys. Rev. B **65**, 134522 (2002).
- ²² T. Gramespacher and M. Büttiker, Phys. Rev. B **61**, 8125 (2000).
- ²³ S.V. Naidenov, and V.A. Khlus, Low Temp. Phys. **21** 462, (1995).
- ²⁴ L.S. Levitov, H. W. Lee. and G.B. Lesovik, J. Math. Phys. (N.Y) **37**, 4845 (1996).
- ²⁵ W. Belzig and Yu. V. Nazarov, Phys. Rev. Lett. **87** 197006 (2001).
- ²⁶ W. Belzig, cond-mat/0210125.
- ²⁷ P. Samuelsson and M. Büttiker, condmat/0207585.
- ²⁸ D. Quirion, C. Hoffmann, F. Lefloch, and M. Sanquer, Phys. Rev. B **65**, 100508 (2002).
- ²⁹ A.F. Volkov, A.V. Zaitsev and T.M. Klapwijk, Physica C, **203** 267 (1993).
- ³⁰ Yu. V. Nazarov, Superlattices Microstruct. **25** 1221 (1999).
- ³¹ This property makes the model relevant also for a chaotic dot with strong tunnel barriers, see e.g. A.A. Clerk, P.W. Brouwer, and V. Ambegaokar, Phys. Rev. B **62**, 10226 (2000).
- ³² Finite frequency properties of NS-systems has been studied in e.g. A.V. Lebedev and G.B. Lesovik, JETP Lett. **74** 570 (2001) and J. Torres, Th. Martin and G.B. Lesovik, Phys. Rev. B **63**, 134517 (2001). The screening properties has been investigated by S. Pilgram, H. Schomerus, A. M. Martin, and M. Buttiker, Phys. Rev. B **65**, 045321 (2002).
- ³³ Yu. V. Nazarov, Ann. Phys. (Leipzig), **8**, Special Issue, SI-193 (1999).
- ³⁴ We always have e.g. the solution $\check{G} = 1$, which has no physical meaning.
- ³⁵ For recent reviews, see e.g. C.J. Lambert and R. Raimondi, J. Phys. Cond. Mat.**10** 901 (1998) and W. Belzig, F.K. Wilhelm, C. Bruder, G. Schön and A.D. Zaikin, Superlattices Microstruct. **25** 1251 (1999).
- ³⁶ W. Belzig, J. Börlin, C. Bruder and Yu. V. Nazarov, cond-mat/0210126.
- ³⁷ B. Reulet, D.E. Prober and W. Belzig, condmat/0210069.